Proceedings of DSCC2008
2008 ASME Dynamic Systems and Control Conference
October 20-22, 2008, Ann Arbor, Michigan, USA

DSCC2008-2216

LMST-BASED SAFETY-PRESERVED CONSENSUS CONTROL OF MULTI-ROBOT SYSTEMS WITH KINODYNAMIC CONSTRAINTS

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ABSTRACT

We report a local minimum spanning tree (LMST)-based consensus control of multi-robot systems. Instead of using a potential function-based approach, we propose a safety region concept for distributed collision-free control system design. The safety region design also takes a consideration of the kinematics and dynamics constraints, namely, kinodynamic constraints of each robot. The network topology control among multiple robots is constructed by an LMST algorithm. The LMST-based topology control not only preserves the connectivity of multi-robot systems but also improves the energy consumption and network communication quality. Simulation results are presented to validate the proposed control system design.

1 INTRODUCTION

In recent years, there are a significant growth of research in the area of cooperative control of the multi-robot systems; see [1] for a recent survey. Most work of cooperative control of multi-robot systems consider either a kinematic or a dynamic robot model with no constraint on the robot's velocity and acceleration bounds. In this paper, we consider a safety-preserved velocity consensus control of multi-robot systems with kinodynamic constraints [2]. Under the velocity consensus control, each robot achieves a same velocity by communicating and exchange information only with its neighboring robots. The kinodynamic constraints here refer to both kinematic, such as obstacle avoidance, and dynamic constraints, such as velocity and acceleration limits. We consider that each robot has its own physical capability, such as velocity and acceleration bounds. For ground robots and vehicles, for example, the maximum acceleration and deceleration depend on the interaction between the wheels and the ground. Moreover, instead of using flooding communication among all robots, the proposed control algorithm is based on a construction of a local minimum spanning tree (LMST) network topology. The LMST-based network structure preserves the connectivity of the information graph and improves the energy efficiency and communication quality [3].

The potential function approach is widely used as a design methodology for collision avoidance with obstacles or moving agents [4–6]. The velocity and acceleration of moving agents are assumed to be unbounded. Recently, some considerations are given for a cooperative control of a robotic team with bounded accelerations [7]. The cooperative control of multi-robot systems under kinodynamic constraints is different with kinodynamic motion planning. For a motion planning problem, starting and ending positions in 2D or 3D environments with obstacles are typically given [2, 8]. In [9], an optimal collision-free coordination scheme is formulated and solved as a mathematical programming problem for multiple robots with kinodynamic constraints. The approach in [9] is however centralized and thus cannot directly be utilized for distributed control of multi-robot systems in a dynamic environment.

In [10], a set of safety platoon maneuvers are designed based on the safety region between two platoons moving along a highway. The safety regions are collision-free profiles for each vehicle under the limited velocity and acceleration capabilities. The automated vehicles in [10] is constrained in one-dimensional space while for multi-robot systems, the motion is either 2D or 3D. A related topic is the concept of "velocity obstacles" that is first discussed in [11] for the motion planning of mobile robots in a dynamic 2D environment. The velocity obstacle is a first-order method (velocity profile) for obstacle-avoidance motion plan-

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ning maneuvers. The motion planning algorithms are formulated as a graph-searching problem by constructing the collision-free and dynamically constrained regions.

For networked cooperative control, connectivity of the associated information graphs is one of the fundamental requirements to guarantee the convergence of the control systems [12]. Several connectivity-preserved control algorithms have been proposed for cooperative control [7, 13–15]. In [16], proximity graphs are used and analyzed as the information network among mobile agents for a consensus algorithm. From communication efficiency and energy consumption viewpoint, a flooding broadcasting scheme is not ideal. Communication power usage is proportional to broadcasting range and using the maximal transmission power at all times is not cost-effective and energy-efficient. Frequent broadcasting will also interfere and reduce the network and communication capacity and efficiency. Topology control algorithms have recently been proposed to maintain network connectivity while reducing energy consumption and improving network efficiency and capacity [17]. The key idea of topology control is to collaboratively determine the transmission power among network nodes and form a proper neighborhood relation [3].

The work in this paper are inspired from recent development in several areas, such as robotic motion planning in dynamic environments, safety control of automated vehicles, and topology control in sensor networks. The contributions of this paper are threefold. First, we take a different approach to design a collision-free control system for multi-robot systems. We consider to design a safety region associated with each robot. The safety region of a robot is a high-dimensional profile that depends on the relative position and velocity of the robot and its neighbors. Comparing with other collision-free designs, such as the potential function approach, the safety region design incorporates robot's kinodynamic constraints and is scalable. Second, the one-hop LMST-based network topology design reduces node degrees, and thus improves communication energy efficiency. The preserved-connectivity of communication networks among robots is formulated as an optimization problem of the weighting factors of control laws. Finally, our approach can be easily extended to other types of communication topology design, such as the k-connectivity fault-tolerant communications [18] and probabilistic and computational approaches [19, 20].

The remainder of the paper is organized as follows. We discuss the safety region design in Section 2. In Section 3, we present an LMST-based topology control for mobile networked robots. We present the velocity consensus control in Section 4. Simulation results are presented in Section 5, and finally we conclude the paper in Section 6.

2 SAFETY REGION DESIGN

2.1 Two-robot safety region design

We consider an N-robot system in a 2D planar space. We denote the multi-robot system as $\mathcal{R} := \{R_1, \cdots, R_N\}$, where the ith mobile robot is denoted as R_i. We also consider that there are M static or moving obstacles, denoted by $O := \{O_1, \cdots, O_M\}$ in the environment. We assume that each robot is equipped with both communication and sensing capabilities. The communication among robots is through *ad hoc* networks and each robot detects the location of obstacles by on-board sensors.

For each robot R_i, the magnitudes of its velocity $v_i \in \mathbb{R}^2$ and acceleration $a_i \in \mathbb{R}^2$ are bounded by $v_{\max}^{R_i}$ and $a_{\max}^{R_i}$, respectively,

$$\|v_i\| \le v_{\max}^{R_i}, \quad \text{and} \quad \|a_i\| \le a_{\max}^{R_i}, \quad i = 1, \cdots, N. \tag{1}$$

For each obstacle O_j, we also have the velocity and acceleration bounds, $\|v_j\| \le v_{\max}^{O_j}$ and $\|a_j\| \le a_{\max}^{O_j}$, $j = 1, \cdots, M$. Here we consider stationary obstacles as a special case of the moving obstacles with zero velocity and acceleration. We assume that for both robots and obstacles, their velocity and acceleration (deceleration) limits are symmetric, for example, for robot R_i, its acceleration follows in a range of $[-a_{\max}^{R_i},\ a_{\max}^{R_i}]$. Moreover, we consider that robots can freely move along any direction and the acceleration can be applied to any direction as well.

We consider robots R_i and R_j as shown in Fig. 1. Let r_i, v_i, and r_j, v_j denote their position and velocity vectors (in the navigation frame $\mathcal{N}$), respectively. The relative position vector r_{ij} between R_i and R_j is then $r_{ij} := r_j - r_i$ and the directional vector $n_{ij} := \frac{r_{ij}}{\|r_{ij}\|}$. It is straightforward to calculate the relative distance $\Delta r_{ij}(t)$ and the relative velocity magnitudes $\Delta \dot{r}_{ij}(t)$ as

$$\Delta r_{ij}(t) := \|r_{ij}(t)\|, \tag{2a}$$
$$\Delta \dot{r}_{ij}(t) := (v_j - v_i) \cdot n_{ij} = -v_{ji} \cdot n_{ij}, \tag{2b}$$

where $v_{ij} = v_j - v_i = -v_{ji}$.

We assume that both robots and obstacles are considered as circular shapes and also rigid bodies. The radius of R_i is denoted by s_i. Since we consider the relative motion between two robots, we consider robot R_i as a point and the size of robot R_j is of a radius of the Minkowski addition $l_j := s_i + s_j$; see Fig. 1. We define an *unsafe impact* between two robots as follows.

Definition 1. *For two robots R_i (a point-robot) and R_j (with a size of radius l_j) as shown in Fig. 1, an* unsafe impact *between R_i and R_j is said to happen at time t if*

$$\Delta r_{ij}(t) \le l_j, \ \textit{and} \ \Delta \dot{r}_{ij}(t) \le 0. \tag{3}$$

We consider the conditions under which the unsafe impact happens are dependent on the magnitudes of relative position $\Delta r_{ij}(t)$, velocity $\Delta \dot{r}_{ij}(t)$, and the relative velocity direction, namely, the angle $\theta_{ij}(t)$ between v_{ij} and n_{ij}. Therefore, we define a *safety region* as follows.

Definition 2. *A* safety region*, denoted as X_{ij}^S, of two robots R_i and R_j is defined as a set of all triples $(\Delta r_{ij}(t), \Delta \dot{r}_{ij}(t), \theta_{ij}(t)) \subset \mathbb{R}^2 \times \mathbb{S}$ such that the unsafe impact conditions (3) are not satisfied.*

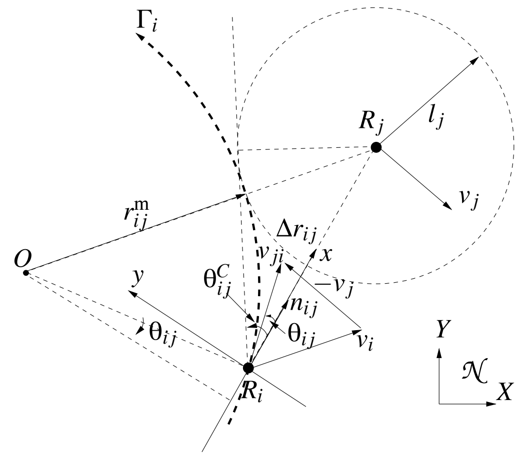

Figure 1. A schematic of two robots R_i and R_j in the 2D space.

To compute the safety region X_{ij}^S, we first find out the direction of acceleration $a_i(t)$ of robot R_i that will maximally change the robot's position along the velocity direction $v_i(t)$. Fig. 2 shows the positions of robot R_i at time t and $t+\Delta t$ (at R_i'). We setup a local coordinate x-y and let the x-axis direction along the velocity $v_i(t)$. Assume the acceleration $a_i(t)$ is along the direction with an angle α with the y-axis direction (Fig. 2). Consider at time $t+\Delta t$, robot R_i is located at R_i' with coordinates $(\Delta x, \Delta y)$ and angle θ with the x-axis direction.

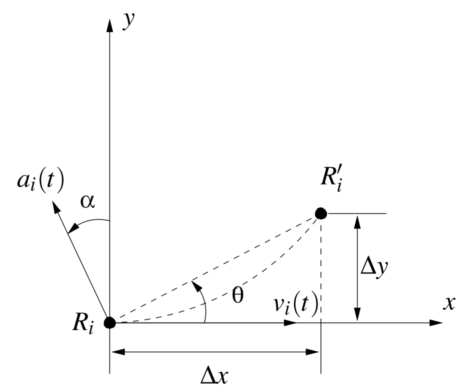

Figure 2. A schematic of two robots R_i and R_j in the 2D space.

With a small Δt, we approximate the position of R_i' as

$$\Delta x = v_i \Delta t - \frac{1}{2}(a_i \sin\alpha)\Delta t^2, \;\; \Delta y = \frac{1}{2}(a_i \cos\alpha)\Delta t^2, \qquad (4)$$

where v_i and a_i are the magnitudes of vectors $v_i(t)$ and $a_i(t)$, respectively. Therefore, we have

$$r_\theta(\alpha) := \tan\theta = \frac{\Delta y}{\Delta x} = \frac{\frac{1}{2}(a_i\cos\alpha)\Delta t}{v_i - \frac{1}{2}(a_i \sin\alpha)\Delta t}. \qquad (5)$$

To make θ to be the maximum at time t, we need $r_\theta(\alpha)$ to be the maximum when $\alpha = \alpha_m$. We take the derivative of $r_\theta(\alpha)$ with respect to α and consider $r_\theta'(\alpha_m) = 0$. Thus, we obtain

$$\sin\alpha_m = \frac{a_i}{2v_i}\Delta t. \qquad (6)$$

From Eq. (6), we consider $\Delta t \to 0$ and obtain $\alpha_m = 0$. Therefore, to make robot R_i change its position as much as possible along the current velocity direction, we need its acceleration $a_i(t)$ is along the direction perpendicular to its velocity direction $v_i(t)$, namely, $a_i(t) \perp v_i(t)$. Furthermore, from Eq. (4), we obtain that $a_i = a_{\max}^{R_i}$ to render Δy to be the maximum. We summarize the above observation as the follow lemma.

Lemma 1. *Suppose robot R_i has velocity and acceleration $v_i(t)$ and $a_i(t)$ at time t, respectively. For a maximum change of its position along the direction of $v_i(t)$, its acceleration $a_i(t)$ should be applied along the direction perpendicular to $v_i(t)$ and its magnitude should be $a_i = a_{\max}^{R_i}$, namely, $a_i(t) \perp v_i(t)$ and $\|a_i(t)\| = a_{\max}^{R_i}$.*

Since we consider the relative motion between R_i and R_j, we assume that robot R_j is stationary and R_j moves with velocity v_{ji}. We setup a coordinate system x-y with the origin at R_i and the x-axis is along the directional vector n_{ij} and the y-axis is perpendicular to n_{ij}. We assume that the relative velocity v_{ji} is at an angle of θ_{ij} with the x-axis direction; see Fig. 1. We define a critical velocity angle $\theta_{ij}^c(t)$ between R_i and R_j as

$$\theta_{ij}^c(t) := \sin^{-1}\left(\frac{l_j}{\Delta r_{ij}(t)}\right). \qquad (7)$$

For robot R_i to avoid a collision with robot R_j, if $\theta_{ij} < \theta_{ij}^c(t)$, robot R_i needs to change its velocity direction. For a safety region definition, we consider an extreme case when robot R_i is about to collide with R_j. From the results in Lemma 1, we need $a_i(t) \perp v_{ji}(t)$ and $\|a_i(t)\| = a_{\max}^{R_i}$ for robot R_i to avoid a collision with R_j for a given relative velocity $v_i(t)$. Therefore, the trajectory Γ_i of robot R_i is a circular curve (centered at point O) that is tangent to the boundary of robot R_j and the relative velocity $v_i(t)$; see Fig. 1. Let r_{ij}^{m} denote the radius of the trajectory Γ_i. From the geometry relationship in Fig. 1, we have the following relationship $\left(r_{ij}^{\mathrm{m}} + l_j\right)^2 = \left(\Delta r_{ij} + r_{ij}^{\mathrm{m}}\sin\theta_{ij}\right)^2 + \left(r_{ij}^{\mathrm{m}}\cos\theta_{ij}\right)^2$. We solve the above equation for r_{ij}^S and obtain

$$\Delta r_{ij}^S(\theta_{ij}) = \sqrt{(r_{ij}^{\mathrm{m}})^2\sin^2\theta_{ij} + l_j^2 + 2l_j r_{ij}^{\mathrm{m}}} - r_{ij}^{\mathrm{m}}\sin\theta_{ij}, \qquad (8)$$

where $r_{ij}^{\mathrm{m}} := \frac{v_{ji}^2}{a_{\max}^{R_i}} = \frac{\Delta\dot{r}_{ij}^2}{a_{\max}^{R_i}\cos^2\theta_{ij}}$. In the above equation, we use the relationship $\Delta\dot{r}_{ij}(t) = v_{ji}(t)\cos\theta_{ij}(t)$. Considering the velocity bounds for robots R_i and R_j, we have the following constraints

$$-v_{ij}^{\mathrm{m}} \le \Delta\dot{r}_{ij} \le v_{ij}^{\mathrm{m}}, \qquad (9)$$

where $v_{ij}^{\mathrm{m}} := v_{\max}^{R_i} + v_{\max}^{R_j}$.

From the above calculation, we can write the safety region X_{ij}^S as

$$X_{ij}^S = \Big\{ (\Delta r_{ij}(t), \Delta \dot{r}_{ij}(t), \theta_{ij}(t)) \subset \mathbb{R}^2 \times S \,\Big|\, \theta_{ij}(t) \geq \theta_{ij}^c(t),$$
$$\text{or } \theta_{ij}(t) < \theta_{ij}^c(t),\ \Delta r_{ij}(t) \geq \Delta r_{ij}^S(\theta_{ij}, t),$$
$$\text{and } \|\Delta \dot{r}_{ij}\| \leq v_{ij}^{\mathrm{m}} \Big\}. \tag{10}$$

It is straightforward to verify from (8) that $\Delta r_{ij}^S(\theta_{ij})$ is a monotonically decreasing function of angle $\theta_{ij} \geq 0$. We therefore consider a special case when the relative velocity $v_{ji}(t)$ is along the direction of the relative position vector $n_{ji}(t)$, namely, $\theta_{ij}(t) = 0$. In such a case, $\Delta r_{ij}^S(\theta_{ij})$ has a minimum value if other variables are fixed. When $\theta_{ij}(t) = 0$, from (8), we obtain

$$\left(\Delta r_{ij}^C\right)^2 - \frac{(v_{ij})^2}{(a_{\max}^{R_i}/2l_j)} = l_j^2, \tag{11}$$

where $\Delta r_{ij}^C := \Delta r_{ij}^S(0) = \Delta r_{ij}^S\Big|_{\theta_{ij}=0}$. Fig. 3 illustrates the intersection region (as the shaded area) of safety region X_{ij}^S and the plane given by $\theta_{ij}(t) = 0$. The boundary of the safety region is given by the hyperbola (11).

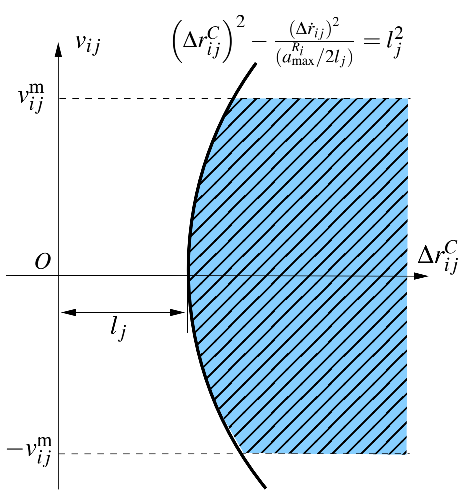

Figure 3. Safety region X_{ij}^S (shaded area) when $\theta_{ij}(t) = 0$.

For a general case when $\theta_{ij} \neq 0$, the safety region is given by Eqs. (8) and (10). Fig. 4 shows an example of the boundary surface of the safety region X_{ij}^S. The safety region is on the upper half volume of the surface shown in the figure. From Fig. 4, we observe that for a larger angle θ_{ij}, the safety region is close to a plane tangent to the circular disk with radius l_j, which implies that robot R_i can get closer to R_j if their relative velocity direction is perpendicular to their relative position direction.

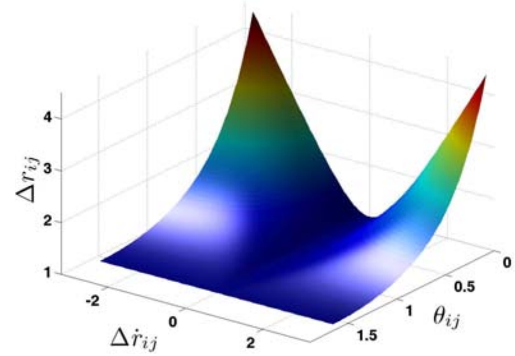

Figure 4. Surface of safety region X_{ij}^S in $\mathbb{R}^2 \times S^1$ for a case of $a_{\max}^{R_i} = 1$ m/s^2 and $l_j = 1$ m/s for $\theta_{ij} \in [0, \frac{\pi}{2}]$..

Before we discuss the safety region of multi-robot systems, we like to discuss a safety relative velocity profile v_{ij}^S given by X_{ij}^S. From Eq. (8), we obtain the relationship between v_{ij} and Δr_{ij}^S as

$$v_{ij}^2 = f_v(\Delta r_{ij}^S) := \frac{a_{\max}^{R_i}\left[(\Delta r_{ij}^S)^2 - l_j^2\right]}{2(l_j - \Delta r_{ij}^S \sin\theta_{ij})}. \tag{12}$$

It is straightforward to check that $\Delta r_{ij}^S \sin\theta_{ij} \leq l_j$ for $\theta_{ij} \in [0, \frac{\pi}{2}]$ and $\Delta r_{ij}^S \geq l_j$ so that the right-hand side of the above equation is non-negative and well-defined. Within X_{ij}^S, when $\theta_{ij}(t) \leq \theta_{ij}^c(t)$, $l_j \leq \Delta r_{ij}(t) \leq \frac{l_j}{\sin\theta_{ij}}$, and the function $f_v(r)$ is a non-decreasing function (we observe this from the fact $f_v'(r) \geq 0$ for $r \in [l_j, l_j/\sin\theta_{ij}]$. We therefore obtain the safety velocity profile v_{ij}^S as

$$v_{ij}^S := v_{ij}^S(\Delta r_{ij}, \theta_{ij}) = \sqrt{\frac{a_{\max}^{R_i}\left[(\Delta r_{ij})^2 - l_j^2\right]}{2(l_j - \Delta r_{ij}\sin\theta_{ij})}}. \tag{13}$$

2.2 Multi-robot safety region

We now discuss the safety region for a case when there are many robots (or obstacles) around robot R_i. We first define a sensing neighboring set $\mathbf{N}_i$ for robot R_i as follows.

Definition 3. *For a set of robots $\mathcal{R}$, the* (sensing) neighbors *of R_i is a subset $\mathbf{SN}_i \subset \mathcal{R} \cup O$ that satisfies $\mathbf{SN}_i = \{R_j \in \mathcal{R} \text{ or } O_j \in O \,|\, \|r_j - r_i\| \leq l_i^s\}$, where $l_i^s > 0$ is the sensing range of robot R_i.*

For robot R_i, the collision-free safety region can be written as the intersection of safety regions of all of its sensing neighboring set, namely,

$$X_i^S = \bigcap_{j \in \mathbf{SN}_i} X_{ij}^S \tag{14}$$

We can prove that the safety region X_i^S defined by (14) is a convex set. The safety region X_{ij}^S is an embedded subspace in $\mathbb{R}^2 \times \mathbb{S}$.

3 LMST-BASED NETWORKS TOPOLOGY CONTROL

We assume that the each robot knows its position and communicates with other robots using *ad hoc* networks. First, we define the physical neighboring set of a robot as follows.

Definition 4. *For a set of robots $\mathcal{R}$, the* (physical) neighboring robots *for R_i is a subset $\mathbf{N}_i \subset \mathcal{R}$ defined as $\mathbf{N}_i = \{R_j \in \mathcal{R} \,|\, \|r_j - r_i\| \le l_i^R\}$, where $l_i^R > 0$ is the maximum communication range of robot R_i.*

Through communication, the robotic team forms an undirected simple graph $G = (V, E)$, where V is the set of robots, and E is the edge set defined by the (physical) neighbors $\mathbf{N}_i$ of each R_i, namely, $E = \{(R_i, R_j) \,|\, R_j \in \mathbf{N}_i\}$. For each robot R_i, we assign a unique id, for example, $id(R_i) = i$, and we denote $G_i = (V_i, E_i)$ as the induced subgraph of G such that $V_i = \mathbf{N}_i$.

We construct the LMST in several steps. First, each node periodically broadcasts a `hello` message using its maximal transmission power to obtain its physical neighbors $\mathbf{N}_i$. Based on $\mathbf{N}_i$, node R_i can construct a local minimum spanning tree $T_i = (V(T_i), E(T_i))$ of G_i which spans all nodes within its neighbors in $\mathbf{N}_i$. The construction of LMST can be obtained by existing algorithms, such as Prim's algorithm [21]. Here a unique weight function (as a triplet) has been defined on the edge (R_i, R_j) as $(\|r_i - r_j\|, \max(id(R_i), id(R_j)), \min(id(R_i), id(R_j)))$ such that the constructed LMST is unique [3]. With the LMST, we define a logical neighboring relationship and logical neighbor set.

Definition 5 ([3]). *Robot R_j is a* (logical) neighbor *of robot R_i, denoted as $R_i \to R_j$, if and only if $(R_i, R_j) \in E(T_i)$. $R_i \leftrightarrow R_j$ if and only if $R_i \to R_j$ and $R_j \to R_i$. The (logical) neighbor set $\mathbf{LN}_i$ of robot R_i is defined as $\mathbf{LN}_i = \{R_j \in V(G_i) \,|\, R_i \to R_j\}$.*

Note that the LMST-based network topology $G_0 = (V_0, E_0)$ has all robots as its node set V_0, and E_0 is constructed by LMST, namely, $E_0 = \{(R_i, R_j) \,|\, R_i \to R_j, R_i, R_j \in V(G)\}$. More detailed can be found in [3]. We define the *degree* of a node as the number of its neighbors. The following properties are obtained from the construction of the LMST and are re-stated here from [3] without proof.

Proposition 1. *The degree of any node in G_0 is bounded by* 6, *namely,* $\deg(R_i) \le 6$, $\forall R_i \in V(G_0)$.

Proposition 2. *The network topology G_0 under LMST preserves the connectivity of G, namely, G_0 is connected if G is connected.*

Proposition 1 implies that the node degree in G_0 is bounded by 6. Indeed, the simulation results in [3] show that an average node degree of 2.06 (compared with 16.48 of all one-to-one communication in $\mathbf{N}_i$) of a randomly distribution of 100 nodes in a 1000×1000 m^2 region and $l_i = 250$ m. The results in Proposition 2 imply that the connectivity of the mobile robotic network is preserved by the LMST topology G_0. This property is important since the convergence of cooperative control strategies is based on the connectivity (or somewhat variations) of the information flow among the networked robots.

Now we consider the mobility of each robot and discuss how the robot's movement affects the local topology G_0. Let ΔT denote the time period of the node broadcasting (`hello` message). We assume that all N robots are distributed in the region with an area S_0. Since robot R_i cannot obtain the kinematics information of other robots outside l_i^R, other robots outside the communication range of R_i are assumed to follow a Brownian-like random motion. Let $n_i := |\mathbf{N}_i|$ denote the number of physical neighbors of R_i, where $|\cdot|$ denotes the cardinality of a set.

We denote the covered communication area of R_i as disk $D(R_i, l_i^R)$. Fig. 5 illustrates a schematic diagram of the probability of a node R_k, which is initially outside of the communication range $D(R_i, l_i^R)$, moves into $D(R_i, l_i^R)$. We denote $r := BC = v_{ik}^{\mathrm{m}} \Delta T > 0$ as the maximum distance between R_i and R_k within ΔT and $x := r_{ik}$. Then the probability p_e that R_k enters disk $D(R_i, l_i^R)$ is illustrated by the shaded area in Fig. 5.

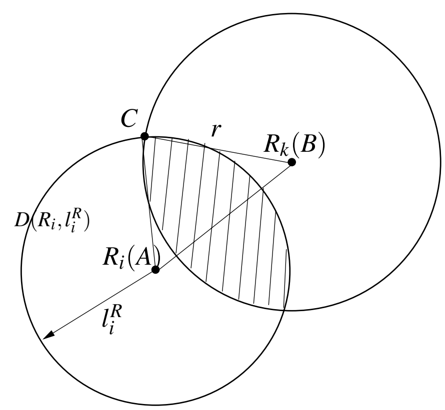

Figure 5. A schematic of calculation of the probability that a new (physical) neighbor R_k moves into the disk $D(R_i, l_i^R)$.

Following the similar treatment in [3], we calculate probability p_e as follows. If $r < 2l_i^R$, then

$$p_e = \int_{l_i^R}^{l_i^R + r} \frac{S_1}{\pi r^2} \frac{2\pi x dx}{S_0} \left(\frac{N - n_i - 1}{N} \right) = \int_{l_i^R}^{l_i^R + r} \frac{2x S_1 \gamma_i}{S_0 r^2} dx,$$

where $S_1 = \alpha_1 (l_i^R)^2 + \alpha_2 r^2 - rx \sin\alpha_2$ is the shaded area, $\alpha_1 := \angle CAB = \cos^{-1}\left(\frac{x^2 + (l_i^R)^2 - r^2}{2x(l_i^R)}\right)$, $\alpha_2 := \angle CBA = \cos^{-1}\left(\frac{x^2 + r^2 - (l_i^R)^2}{2xr}\right)$, and $\gamma_i := \frac{N - n_i - 1}{N}$ is the ratio of the numbers of nodes outside disk $D(R_i, l_i^R)$ and the total robotic nodes.

If $r \geq 2l_i^R$, we similarly obtain

$$
\begin{aligned}
p_e &= \int_{l_i^R}^{r-l_i^R} \frac{\pi (l_i^R)^2}{\pi r^2} \frac{2\pi x dx}{S_0} \gamma_i + \int_{r-l_i^R}^{r+l_i^R} \frac{S_1}{\pi r^2} \frac{2\pi x dx}{S_0} \gamma_i \\
&= \frac{\pi (l_i^R)^2 (r - 2l_i^R)\gamma_i}{S_0 r} + \int_{r-l_i^R}^{r+l_i^R} \frac{2xS_1\gamma_i}{S_0 r^2} dx.
\end{aligned}
$$

We then obtain the expected (or estimated) number of nodes $\hat{n}_i^e = (N - n_i - 1)p_e$ that enter the disk $D(R_i, l_i^R)$ within the time period ΔT.

The number of nodes that leave the disk $D(R_i, l_i^R)$ can be estimated in a deterministic fashion because each node in $D(R_i, l_i^R)$ can broadcast its kinematics information to R_i and, thus, we can approximately determine each neighbor's location assuming its velocity is constant within ΔT. Fig. 6 illustrates such a scenario. At current time, robot R_i has four physical neighbors R_{ij}, $j = 1, \cdots, 4$. After ΔT, each neighbor R_{ij} moves to its new position R'_{ij}. Since robot R_i receives kinematics information from each of its physical neighbors, it can predict the location of each neighbor after ΔT and, thus, estimate the nodes who will move out of disk $D(R_i, l_i^R)$.

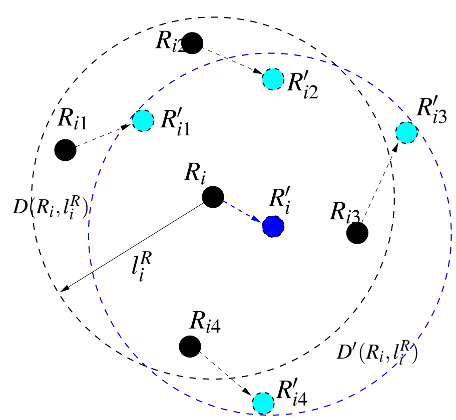

Figure 6. A schematic of calculation of the location (R's) of each neighbor in $D(R_i, l_i^R)$ after ΔT. Each solid circle R_{ij}, $j = 1, \cdots, 4$, indicates the location of neighbors and each dash-line circle indicates the new location of R'_{ij} after ΔT.

Let $\hat{n}_i^l$ denote the estimated number of neighbors of R_i by using the current kinematics information from neighbors in $\mathbf{N}_i$. We consider $\hat{n}_i^l = |\mathbf{L}_i|$ and the set $\mathbf{L}_i$ is denoted the robots that will leave disk $D(R_i, l_i^R)$ within ΔT,

$$
\mathbf{L}_i = \{R_j \in \mathbf{N}_i \,|\, \|\hat{r}_j(t + \Delta T) - r_i(t)\| > l_i\},
$$

where $\hat{r}_j(t + \Delta T) := r_j + v_j \Delta T$ is the estimated position vector of R_j in $\mathbf{N}_i$. For example, consider the scenario shown in Fig. 6, $\mathbf{L}_i = \{R_{i3}, R_{i4}\}$ and therefore $\hat{n}_i^l = 2$. With the estimation of the number of nodes that are entering and leaving the disk $D(R_i, l_i^R)$, we obtain the estimated net change of the node number

$$
\Delta n_i = \hat{n}_i^e - \hat{n}_i^l. \tag{15}
$$

4 COLLISION-FREE CONSENSUS CONTROL

In this section, we present a collision-free consensus control of multi-robot systems in a dynamic environment. The control system design is based on the safety region that is discussed in Section 2 and the LMST-based network topology presented in the previous section.

We consider a discrete-time particle dynamics model for each mobile robot at time k,

$$
\begin{cases}
r_i(k+1) = r_i(k) + v_i(k)\Delta T_c, \\
v_i(k+1) = v_i(k) + u_i(k)\Delta T_c,
\end{cases} \tag{16}
$$

where $u_i(k)$ is the controlled acceleration (at the kth step) and ΔT_c is the control updating time period. For robot R_i, we consider the following velocity control law

$$
u_i(k) = \sum_{j \in \mathbf{LN}_i} a_{ij}(k)(v_j(k) - v_i(k)), \tag{17}
$$

where $a_{ij}(k) > 0$ are the weighting factors. If we define $a_{ii}(k) = 0$, $i = 1, \cdots, N$, then the matrix $A(k) = [a_{ij}(k)]$ can be considered as a *weighted adjacency matrix* of LMST G_0. Note that due to the mobility of robots, matrix $A(k)$ is time-varying. We also consider the constraint $\sum_{j=1}^{|\mathbf{LN}_i|} a_{ij} = 1$, $a_{ij} > 0$ for a scaled velocity distribution among robots.

The consensus control law (17) is similar to those in [5,22]. The only difference here is that we consider the LMST G_0 as the information network while most other control laws assume one-to-one communications within the maximal transmission range. Based on the control law (17), we have to consider the following three requirements:

R1. Collision avoidance. We have to consider the collision among robots and between robots and (stationary or moving) obstacles;
R2. Preserved connectivity among the robot network; and
R3. Each robot's motion has to satisfy the dynamic constraints, namely, with velocity and acceleration bounds.

To satisfy the above requirements, we consider to optimize the weighting factors $a_{ij}(k)$ over the safety region X_i^S and to consider the topology change among the physical neighbors of robot R_i. Note that the safety region X_i^S is obtained by considering the kinodynamic constraints and is collision-free. By choosing an appropriate set of a_{ij}, we may possibly control the velocity v_i

in X_i^S and, thus, satisfy Requirements R1 and R3. To satisfy Requirement R2, we consider an optimization problem to maximize the net change of the neighbor nodes of R_i as follows.

$$
\begin{aligned}
a_{ij}^*(k) &= \arg\max_{a_{ij}(k)} \Delta n_i(k) \\
\text{s.t.} \quad & v_i(k) \in X_i^S(k),\ |u_i(k)| \le a_{\max}^{R_i} \\
& \sum_j a_{ij}(k) = 1,\ a_{ij}(k) > 0. \qquad (18)
\end{aligned}
$$

Intuitively, if the number of nodes in the neighbor set $\mathbf{N}_i$ does not decrease over time, then by the construction of LMST, the connectivity of the network among mobile robots is guaranteed. Under such a design, we have the following consensus control performance.

Theorem 1. *If there exists a set of a_{ij} by (18) such that the number of neighboring nodes is kept non-decreasing (i.e. $\Delta n_i(k) \ge 0$), then under the consensus control law (17), the robot team $\mathcal{R}$ converges to the same velocity ($v_i \to v^*$ as $t \to \infty$, $\forall i$, where v^* is determined by the initial configuration of the networks and robots' velocities) while satisfying the kinodynamic constraints.*

Proof. The convergence of the consensus control (17) comes from results in [22] if the network topology is connected. The connectivity of the robot network is from the construction and properties of LMST. If the neighboring node number of R_i is non-decreasing ($\Delta n_i(k) \ge 0$), then by the LMST construction, we can guarantee both connectivity of the communication network and kinodynamic constraints by (18). This completes the proof.

It is noted that we cannot guarantee that there always exist solutions of a_{ij}s in (18). There is a trade-off among safety robot maneuvers under kinodynamic constraints, dynamically changing topology among the robotic team, and connectivity among robots.

5 SIMULATION RESULTS

In this section, we demonstrate the proposed control design through simulation examples. We consider a scenario where 30 robots are randomly placed on a 20 m × 20 m square; see Fig. 7(a). The initial velocities along the x- and y-axis directions for each robot are assigned randomly between 1 m/s and 2 m/s, respectively. Each robot is assumed to have the same circular size with radius $s_i = 0.1$ m. The maximum velocity and acceleration for each robot are $v_{\max}^{R_i} = 2$ m/s and $a_{\max}^{R_i} = 0.1$ m/s^2.

Figure 7(b) shows the flooding-communication topology at the beginning of the simulation. The initial LMST topology is shown in Fig. 7(c) and the trajectory of the robot team under the velocity consensus control is illustrated in Fig. 7(d).

The trajectory demonstrates that the velocities of all robot converge to the same value. To see that clearly, Figure 8 shows the velocity of each robot in the X- and Y-axis directions. It is clearly observed that the convergence of the robot's velocities.

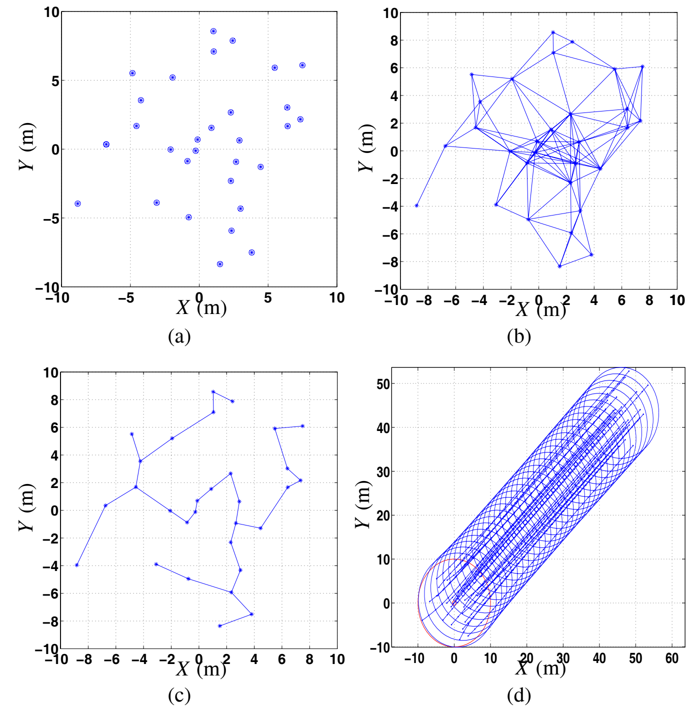

Figure 7. (a) A randomly distributed 30-robot team on a square at the initial time. (b) Initial one-to-one communication topology. (c) Initial LMST topology. (d) Trajectory of the robot team.

Figure 9 shows a few snapshots of the LMST topology of the robot team during the simulation period. From these snapshots, we clearly observe that the topology of the LMST varies when the robots move during the simulation. Since the LMSTs are always connected, the convergence results are obtained. These simple simulation results demonstrate the proposed algorithms.

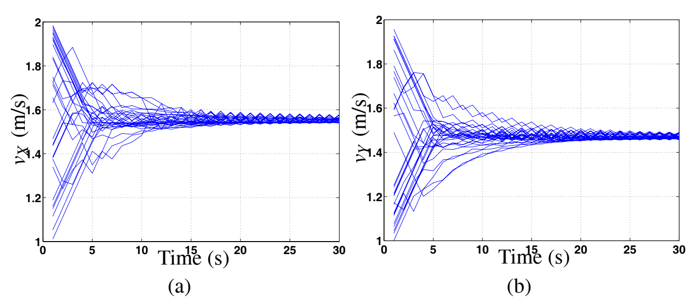

Figure 8. Velocity profiles of each robot. (a) Velocity along the x-axis direction. (b) Velocity along the y-axis direction.

6 CONCLUSION

In this paper, we developed an LMST-based consensus control of multi-robot systems. We proposed a safety-region concept for distributed collision-free control system design. Comparing with the potential function-based collision-free approach,

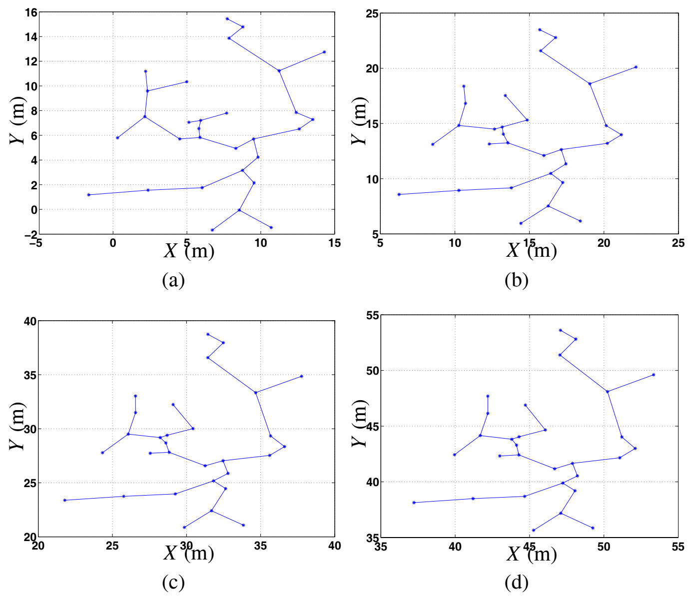

Figure 9. Snapshots of the LMST topology. (a) $t = 5$ s. (b) $t = 10$ s. (c) $t = 20$ s. (d) $t = 30$ s.

the proposed scheme provides a safety region-based on the kinodynamic constraints. The network topology is constructed by an LMST topology among multi-robot and therefore the communication overhead can be reduced significantly comparing with the flooding-communication topology. A connectivity-preserving movement is designed for each robot based on the LMST-based topology. Preliminary results have demonstrated the efficiency and effectiveness of the proposed control system design. Currently, we are refining the control design and testing more complex scenarios is also ongoing research.

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